Math 10460 - Honors Mathematics II
Homework 4b - Due Wednesday, February 10

Let $d > 0$, and $\varepsilon \geq 0$. The polar equation

$$r = \frac{d}{1 + \varepsilon \sin\theta} \qquad (*)$$

is also a conic section, and it fits the same rule as before

- $\varepsilon = 1$ is a parabola
- $\varepsilon \geq 1$ is a hyperbola
- $0 \leq \varepsilon < 1$ is an ellipse

Please do the following exercises:

(1) Verify the facts that

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$$
$$\sin\left(\theta - \frac{\pi}{2}\right) = -\cos\theta$$
$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$$

and

$$\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$$

(2) (a) Graph $r = \cos\theta$ and $r = \cos\left(\theta - \frac{\pi}{4}\right)$ on the same set of axes. What do you see?
(b) Find Cartesian equations for both graphs.
(c) What are the radii of the two circles?
(d) What are the centers of the two circles?
(e) What is the distance from each center to the origin?
(f) What can you now *conclusively* say about the graph of $r = \cos\left(\theta - \frac{\pi}{4}\right)$ in relation to the graph of $r = \cos\theta$?

(3) Consider the graph of the polar function $r = f(\theta)$ and let ω be some fixed angle. How does the graph of $r = f(\theta - \omega)$ relate to the original graph if $\omega > 0$? if $\omega < 0$? Give some justification in each case.

(4) Find Cartesian coordinates for the three types of conic sections which are possibilities for $(*)$. *Hint*: The solution to the previous problem can make this much easier than repeating the analysis of section 12.2 of the polar packet.